Informal Groundwater Markets:

The Role of Share Contracts

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Abstract

Many small farmers in developing countries have to rely on so-called 'informal groundwater markets' when they need additional water to irrigate their fields. In these markets, groundwater is traded either against a fixed rate per unit, or against a share of the farmer's crop output. I study the bargaining over the terms of a groundwater contract between a farmer and a wellowner and show that the choice of a sharecropping arrangement or a fixed rate depends on the degree of risk aversion of the contracting parties as well as on external factors such as the distribution of the amount of rainfall.

1 Introduction

It is a well-documented fact that informal markets for groundwater are active in a large part of the Indian agricultural sector[1]. These informal groundwater markets are normally found to emerge in situations where the traditional surface water irrigation systems such as tank irrigation fail to cover the water needs of all cultivating farmers. The good traded in these informal markets is groundwater extracted from the soil by farmers owning wells and pump-sets[2]. In exchange for his groundwater the respective wellowner receives either a share of the crop output produced with his water or a fixed payment per hour of irrigation, that is, one has a similar payment structure as it is often observed in the context of land transactions in rural areas.

So far, there is much empirical, but little theoretical work on informal groundwater markets. In many of the empirical papers cited above it is argued that informal groundwater markets may be a suitable institution for supplying small farmers who are not able to invest in an own well with necessary irrigation. This literature, however, also argues that the terms of such water contracts are often exploitative, especially when they take the form of sharecropping arrangements. Turning to the two existing theoretical papers on this issue, Jacoby et al. (2001) investigate price discrimination and monopoly power in informal groundwater markets in Pakistan's southern Punjab, testing whether tubewell owners price-discriminate between their

[1] See for example Saleth (1998), Meinzen-Dick (1998), Satyasai et al. (1997), Janakarajan (1993), Shah (1991).

[2] There is an open access regime for the groundwater resource, that is, every farmer who owns the necessary extracting facilities can appropriate as much groundwater as he wants. This issue will not be considered in this paper.

own share-tenants and other cultivators, and whether tubewell owners who face only the marginal costs of extraction as their shadow price of groundwater use more groundwater per acre on their own plots than their tenants and their other buyers use on their own plots. Since both price discrimination as well as different groundwater input intensities are found to be prevalent in the data, the authors conclude that there is evidence for monopoly power on the part of the tubewellowner, but they also find that monopoly power in the groundwater market has only limited effects on efficiency and equity. They do not address the issue of different contract forms in the groundwater market. Kajisa and Sakurai (2000) explore theoretically as well as empirically the individual-level determinants of groundwater prices using a bilateral bargaining framework and data from six villages in Madhya Pradesh, India. In their analysis, they take into account the fact that there are different payment modes and investigate whether the contract form has an effect on the price per unit of groundwater. They find that the price per unit of groundwater under a share contract will normally be higher than the unit price under an arrangement including a fixed payment due to a risk premium paid to the water seller for shouldering part of the production risk. They do not, however, address which are the determinants of the choice of the contract form, i.e. they do not answer the question why both contract forms coexist in the same groundwater market.

On the other hand, the theoretical literature on contract choice in the related context of tenancy is extensive. Among the different approaches taken to explain the existence of sharecropping arrangements in the tenancy market, the present paper is most closely related to the contributions which use a bargaining approach to explain the parameter values of individual contracts[3], i.e. the papers by Bell and Zusman (1976), Zusman and Bell (1989) and Quiggin and

[3] A discussion of bargaining solutions in the context of rural contracts is given in Bell (1989).

Chambers (2001), rather than to assume that the terms of contracts are taken by all agents as given price-like parameters, as it is done for example in Bardhan and Srinivasan (1971) or in Newbery (1977), or that there is a principal-agent structure where the principal has all the bargaining power, and is consequently in a position to dictate the terms of the contract, as for example in Eswaran and Kotwal (1985) and Stiglitz (1974). In the context of informal groundwater transactions it seems reasonable to assume that the terms of the groundwater trade are the result of bilateral bargaining between the buyer and the seller. Empirical evidence suggests neither that all bargaining power rests with the water seller, which would permit the latter to set the terms of the contract, nor that complete markets for groundwater exist in which all participants take the price as given. For example, the bargaining power of each party depends on the amount of water available from the surface irrigation system in the period of the contract, on the expected amount of rainfall, and on the number of other potential buyers or sellers. If, moreover, both contracting parties are wellowners, bargaining power also depends on whether the seller of the present period's contract is a potential buyer in the next period. Finally, if two well-owning farmers contract, both of them may simultaneously be a seller and a buyer if each of them owns a field plot which cannot be irrigated with his own well but lies within reach of the other party’s well.

In the context of tenancy, Bell and Zusman (1976) consider risk-neutral landlords and tenants and use the Nash-Bargaining-Solution and the assumption that there are non-tradable production factors to explain the existence of sharecropping, but in their analysis the agents cannot choose between sharecropping and fixed-rent contracts. In a framework of pairwise-bargained agency contracts between risk-averse landlords and tenants, Zusman and Bell (1989) derive in an illustrative example a result for the optimal cropshare, which is similar to that derived by me, under the assumption that there is no moral hazard. Consequently, no incentive problems arise. In another illustrative example, where incentive problems are introduced, the result for the optimal cropshare diverges from mine.

In the present paper, the amount of the crucial input, groundwater, is assumed to be observable but not enforceable. Consequently, an incentive problem arises.[4] Based on the author's own empirical observations in a south Indian village, the present paper addresses the question which factors determine the choice of the terms of trade in informal groundwater transactions between a wellowner and a farmer who is in need of additional water for irrigation. I will particularly be concerned with establishing under what circumstances a share contract is chosen rather than a fixed payment per hour.

I model the process of contracting between a risk averse wellowner who faces constant marginal costs for the extraction of groundwater and a risk averse farmer who is in need of additional irrigation as a three-stage game. At the first stage, during the growing season, both parties observe the state of the farmer's crop and bargain over the contractual parameters, namely, a cropshare, a fixed payment per unit of water, and an unrestricted transfer payment[5]. In the second stage, nature chooses the amount of rainfall, which is observed by both parties. In the third stage, the wellowner chooses the amount of groundwater he wants to apply to the farmer's crop, where the amount of groundwater the wellowner delivers can be observed by the farmer, but the farmer cannot compel the wellowner to deliver a certain amount of water. I show that the contractual parameters are chosen in such a way that the wellowner always chooses the efficient amount of groundwater and that there is always efficient risk sharing, regardless of incentive

[4] Stiglitz (1974) considers the trade off between risk sharing and incentives.

[5] Another paper in which contracts with three contractual parameters are considered, is Laffont/Matoussi (1995).

considerations. If the utility functions of both the buyer and the seller exhibit constant absolute risk aversion, the optimal cropshare as well as the optimal fixed payment per unit of water are functions of the coefficients of absolute risk aversion of the two parties. For the case where both parties have utility functions of the logarithmic type, I cannot derive an explicit solution, but I can show how the contractual parameters are influenced by factors such as the buyer's and the seller's incomes from sources other than the contract, the marginal costs of producing the groundwater, and the distribution function of the amount of rainfall in the production period.

The present paper makes also a contribution to the literature on cost-sharing arrangements in the context of sharecropping contracts. Braverman and Stiglitz (1986) show that the resolution for the seeming paradox of the irrelevance of cost-sharing is an asymmetry of information concerning the optimal input use between the landlord and the tenant. Under the assumptions of our model, in contrast, cost-sharing arises because of the risk aversion of the contracting parties combined with the unenforceability of the input in question, whereby cost-sharing is feasible because the supply of the respective input is observable by both parties.

The remainder of the paper is organized as follows: In section two I give a detailed description of the structure of informal groundwater transactions, in section three the model and the results are presented, and section 4 concludes the paper.

2 The structure of informal groundwater transactions

To motivate the structure of the model to be presented in section 3, I first present some empirical facts on informal groundwater 'markets' on the village level. Most of the information presented here comes from a field study conducted by the author in four villages in Nanguneri District, Tamil Nadu, India during January and February 2001. For a more detailed discussion see Steinmetz (2001).

An important point to be made at the beginning is that it is somewhat misleading to use the term 'market' (as normally defined) in this context, because what is observed in reality are personalized contracts between two parties where often both of them have only few or no alternative partners to contract with, rather than an arrangement for transacting in a clearly defined good which is traded at a common price, and with the possibility for each buyer to contract with each seller in the market and vice versa.

The reason for informal groundwater transactions to arise is the undersupply of irrigation water from common sources such as rivers or rainfed reservoirs (in south India known as 'tanks'), combined with the fact that not all farmers are owners of a well and a pumpset. In villages where water is rather scarce, it is normally the case that at some moment in time during the crop season the irrigation water available from the rainfed tank will be used up, so that farmers have to rely on two other irrigation sources to complete their cultivation: rainfall and groundwater. A farmer who does not own groundwater-extracting facilities or whose well does not deliver enough water either has to rely entirely on rainfall, which is random in timing and amount, so that it may happen that the crop fails because of a lack of water, or he has to buy groundwater from another wellowner. In most cases, there will be only a few wellowners, or even only a single wellowner from whom a farmer can buy groundwater because the terrain and the distance between the farmer's field and the well impose technical restrictions on the trading of groundwater.

Empirically, a typical crop and contracting cycle look as follows: At the beginning of the crop season, a farmer starts cultivation, knowing how much water for irrigation purposes he will get from the common tank, but not knowing if there will be rainfall again later in the season to complete cultivation without buying additional water. As the season proceeds, the farmer waits for rain as long as it is possible without incurring the risk of a crop loss before he approaches a wellowner and attempts to bargain over the terms of the water sale. If they agree on a cropsharing arrangement, the area to be irrigated comes entirely under the control of the wellowner, who now decides how much water will be applied to the crop; if, however, a fixed rate per hour of irrigation is chosen, the field remains under the management of the farmer himself, who has to turn to the wellowner for every additional irrigation. An important point to note in this context is that in the case of a share contract, the wellowner always gets his share of the crop, even if he chooses not to apply a drop of water (because of sufficient rainfall) after the contract was agreed on. If, on the other hand, the contract is based on a fixed payment per hour of irrigation, the wellowner is paid only if he delivers the water. The situation is normally such that the farmer can observe the amount of water the wellowner delivers to his field, but that he cannot force the wellowner to deliver a certain amount of water because the wellowner may need the water for his own crop, he may have another higher yielding alternative use, or simply because his pumpset breaks down.

Turning to the question which factors empirically determine the choice of the payment mode, I find evidence (see chapter 1) that the point in the cropping season at which the farmer approaches the wellowner offer of a contract plays an important role: If the contract is chosen early in the season, then, in most cases, both parties agree on a sharecropping arrangement, whereas if the farmer contacts the wellowner late in the season, asking only for a few irrigations, an agreement on a fixed payment per hour of irrigation is more likely to be the outcome of the bargaining. There are two immediate explanations for this. First, the farmer is liquidity-constrained and cannot afford to pay in cash for the obligations resulting from 60 days of groundwater irrigation. Secondly, he is risk averse: if the provision of groundwater irrigation has the effect that output becomes less random or even non-random, then a risk averse farmer is likely to prefer a share contract if he needs groundwater irrigation for a long period, because in the case of a share contract, the payments he has to make do not vary much across the states of nature, whereas in the case of a fixed payment per hour, he has to pay much more if there is low rainfall than when there is high rainfall. That is to say, if a farmer pays a share of his output instead of a fixed rate, he can avoid severe income shocks in a situation where he has to buy groundwater for a long period. With this intuition in mind, I proceed in the next section with the exposition of the model.

3 The Model

In this section, I first set up a general model which enables us to derive some results which hold for a wide variety of cases. I then proceed by specifying the utility functions of the buyer and the seller, and the distribution of the amount of rainfall in order to make some statements concerning the factors influencing the contractual parameters. In the third subsection I briefly examine the case of only two contractual parameters.

3.1 The General Model

To model the process of contracting between a cultivating farmer and a wellowner described above, I assume that an output, q, is produced with only one input, namely water, where the total amount of water, W, is the sum of the amount of rainfall, R, in the period under consideration and the amount of groundwater, G, bought from the wellowner. In effect, I implicitly assume that all other inputs necessary in production, such as land, labour, canal water, and fertilizer, have already been chosen by the farmer by the time he has to decide whether to enter a contract with a wellowner or not. This implies, in particular, that I do not model the farmer's decision of how much land to cultivate, although this decision will certainly be influenced by the effects it will have on the terms of the groundwater contract. I also assume, for simplicity, that neither the farmer nor the wellowner have other parties to contract with. This is not an unrealistic assumption, since there are technical restrictions on trade in groundwater in form of the impossibility of transporting water over large distances. It also excludes the possibility that either the seller or the buyer derive their bargaining power from the number of alternative contract partners they face. The bargaining positions in this case will depend only on the expected amount of rainfall. Let the production function $f(W)$ be everywhere increasing, twice differentiable, concave, and satisfy the lower and upper Inada-conditions. Thus, I assume that there is never too much rainfall. This rather restrictive assumption seems to be justifiable if one considers drought-prone areas such as the Nanguneri District in southern Tamil Nadu, where flood-like conditions and crop damaging rainfalls outside the monsoon period are a very unlikely event. Concerning the distribution of rainfall in the production period under consideration, I assume that there is a continuous distribution of rainfall on the interval $[0,\overline{R}]$, with a differentiable distribution function $H(R)$ and density function $h(R)$. Turning to the wellowner, I assume that the cost of extracting one unit of groundwater is constant and equal to c, and that there is no cost for transporting the groundwater from the well to the farmer's field. I will also neglect any problems caused by the overexploitation of groundwater, although in reality this is an important cost factor which farmers should take into account to guarantee an intertemporally efficient allocation of groundwater.

Let both the buyer and the seller be risk averse with twice differentiable, concave utility functions defined over income, $v(Y^B)$ and $u(Y^S)$ respectively. I assume further that both the buyer and the seller have a perfectly certain source of income, yielding say $\overline{Y}^B$ and $\overline{Y}^S$, the level of which is such that both parties can always fulfil their obligations arising from a contract. For the farmer, $\overline{Y}^B$ is the non-negative income that he derives from some other occupation than farming, and that is always sufficient to cover both non-water inputs and the payments for water that must be made before the harvest is in. This assumption could be problematic since in real world scenarios the wealth (credit)-constraints that a farmer faces seem to have a significant influence on the choice of the contract form.

I model the process of contracting between the cultivating farmer and the wellowner as a three-stage game, assuming that the farmer can observe the amount of groundwater the wellowner delivers to his field, but that he cannot compel the wellowner to provide a certain amount of water, that is, the amount of groundwater to be applied to the crop cannot be stipulated in the contract. This non-enforceability of the input groundwater could arise, for example, because the wellowner has an uncertain alternative use for his groundwater that I do not model here. In the first stage both the farmer and the wellowner observe the state of the crop before they bargain over the contractual parameters, i.e. the wellowner's share of the crop output α with $\alpha \in [0,1]$, a fixed payment per unit of groundwater β paid by the farmer (with $\beta > 0$), and an unrestricted transfer payment γ. There is no such explicit transfer payment observed in reality, but I use γ as an instrument to isolate the effects which only result from the necessity to transfer wealth. In real world contracts there are sometimes implicit transfer payments, for example, labour services done by the farmer on the wellowner's fields.[6] In the second stage, after the

[6] Braverman and Stiglitz (1986) also discuss whether or not to include an unrestricted transfer payment in the theoretical analysis (p. 648): "There is some debate about whether this case ($\gamma = 0$), or the case described in the next subsection, where γ is set optimally, is the more relevant. Observed contractual relationships seldom seem to involve fixed transfers between the landlord and the tenant. On the other hand, there are several contractual provisions which may serve as a substitute; for instance, if the landlord provides a certain minimal level of the input, *x*,

farmer and the wellowner have agreed on a contract form, nature chooses the amount of rainfall. In the third stage the wellowner observes the amount of rainfall, and then decides how much groundwater he wants to apply to the crop, that is, the wellowner chooses the amount of his input under certainty.

I solve the game described above by backward induction, employing the Nash Bargaining Solution to model the bargaining process in the first stage. I begin with the third stage where the wellowner chooses the amount of groundwater he wishes to apply, that is, he maximizes his income $Y^S = \alpha f\left(R+G\right)+\left(\beta - c\right)G+\gamma+\overline{Y}^S$ with respect to the amount of groundwater to be supplied:

$$\max_G \alpha f\left(R+G\right)+(\beta-c)G+\gamma+\overline{Y}^S \tag{0.1}$$

which yields the first-order condition

$$\alpha f' + \beta - c \leq 0, \qquad G \geq 0. \tag{0.2}$$

Let us assume that there is always an interior solutions to (0.2), that is, $\alpha f' + \beta - c = 0$ for all possible values of α, β, and c. From this first-order condition it is then immediately clear that the optimal groundwater contract has to be always such that $\beta < c$ since otherwise there will be unlimited demand for *G*.

Define $G^o\left(R\right) \equiv \arg\max_G Y^S$ as the amount of groundwater which maximizes the wellowner's income in each state of nature $R \in \left[0, \overline{R}\right]$. Then, from (0.2), I have

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it is equivalent to $\gamma < 0$; or if the tenant is required to purchase certain inputs from the landlord at above market prices, it may be equivalent to a contract with $\gamma > 0$."

$W^o(\alpha,\beta;R) = G^o(\alpha,\beta;0) = R + G^o(\alpha,\beta;R)$, that is, the wellowner chooses the amount of groundwater such that in each state of nature the same amount of water is applied to the crop, given a set of contractual parameters (α,β). From this it follows immediately that $f(W^o) = f(R + G^o)$, which means that the potential output uncertainty caused by stochastic rainfall is in fact eliminated by the wellowner's choice. This result is a consequence of the assumption that the cost of producing one unit of groundwater is constant and not random.

It is interesting to note that the variability of the total amount of water the crop receives depends crucially on the behavior of the marginal cost function given a certain distribution of rainfall. Consider the case where, in contrast to our model, the wellowner faces increasing marginal costs in extracting the groundwater, i.e. $c'(G^o) > 0$ and $c''(G^o) > 0$. Then I can derive from the resulting first-order condition for an optimal amount of groundwater the comparative static result: $\frac{\partial G^o}{\partial R} = -\frac{f''}{f'' - c''} < 0$. In our case with constant marginal extraction costs, the above expression becomes $\frac{\partial G^o}{\partial R} = -1 < 0$. It is apparent that $\frac{f''}{f'' - c''} < 1$ for all $c'' > 0$. Thus, in the case of constant marginal extraction costs, a change in the state of nature is completely offset by the change in the amount of groundwater chosen by the wellowner, whereas in the case of increasing marginal extraction costs, a change in the amount of rainfall is compensated for by the wellowner by less than the full amount. That is, in the case of constant marginal extraction costs, the variance of the total amount of water applied to the crop is zero, whereas in the extreme case of infinitely increasing marginal extraction costs, the variance of the total amount of water is the same as the variance of the rainfall distribution, since $\lim_{c'' \to \infty} \frac{\partial G^o}{\partial R} = 0$. The less convex are the extraction costs, the closer to one is $\left|\frac{\partial G^o}{\partial R}\right|$, and the smaller is the variance of the resulting distribution of the total amount of water the crop receives. Since in the following I consider only the case of constant marginal costs, I cannot draw further conclusions from this result, but there is an obvious link to the empirical observations: In areas where the wellowners face stronger increasing marginal extraction costs because the groundwater table in their wells decreases more rapidly during the season, cropsharing contracts are found to be very common. If the ex-ante total amount of water is more risky, and therefore the ex-ante crop output is also more risky, then a risk averse farmer is likely to prefer a payment mode which transfers part of the production risk to the wellowner.

Standard comparative static analysis yields

$$\frac{\partial W^o}{\partial \alpha} = -\frac{f'}{\alpha f''} > 0, \quad \frac{\partial W^o}{\partial \beta} = -\frac{1}{\alpha f''} > 0 \tag{0.3}$$

Now consider the bargaining process at the second stage. If the farmer and the wellowner come to an agreement, the farmer's state-dependent income is $Y^B = (1-\alpha) f\left(R + G^o\right) - \beta G^o - \gamma + \overline{Y}^B$; if there is no agreement, and the farmer has to depend entirely on rainfall, his expected disagreement payoff is $E\left[v\left(f(R) + \overline{Y}^B\right)\right]$. For the wellowner, I assume that in each state of nature he only earns his certain income $\overline{Y}^S$ if there is no contract with the farmer, that is his expected disagreement payoff in utility terms is $u\left(\overline{Y}^S\right)$.

The outcome of the bargaining process is modelled by choosing (α, β, γ) to maximize the product of the gains from cooperation, expressed in terms of expected utility:

$$\max_{\alpha,\beta,\gamma} \Delta\widetilde{V}\left(Y^B\right)\Delta\widetilde{U}\left(Y^S\right) \tag{0.4}$$

$$\text{s.t.} \qquad \alpha \in [0,1], \qquad \beta \geq 0,$$

where

$$\Delta\tilde{V}\left(Y^B\right) = E\left[v\left((1-\alpha)f\left(W^o\right) - \beta G^o\left(\alpha,\beta;R\right) - \gamma + \bar{Y}^B\right)\right] - E\left[v\left(f\left(R\right) + \bar{Y}^B\right)\right] \quad (0.5)$$

and

$$\Delta\tilde{U}\left(Y^S\right) = E\left[u\left(\alpha f\left(W^o\right) + (\beta - c)G^o\left(\alpha,\beta;R\right) + \gamma + \bar{Y}^S\right)\right] - u\left(\bar{Y}^S\right) \quad (0.6)$$

To simplify notation, in the following I will write G^o instead of $G^o\left(\alpha,\beta;R\right)$. It is important to note that the total amount of water applied to the crop in the optimum, W^o, is not random, only the optimal amount of groundwater chosen by the wellowner, G^o, is random. Thus, in the following expectations are taken with respect to G^o. This non-randomness of W^o enables us to draw some terms, especially the comparative static expressions $\frac{\partial W^o}{\partial \alpha}$ and $\frac{\partial W^o}{\partial \beta}$, out of the expectations operator, as it is done for the following first-order conditions.

The first-order conditions for the maximization problem are:

$$\left(-f + \frac{\partial W^o}{\partial \alpha}\left((1-\alpha)f' - \beta\right)\right)E\left[v'\right]\Delta\tilde{U}\left(Y^S\right) + fE\left[u'\right]\Delta\tilde{V}\left(Y^B\right) \leq 0, \qquad \alpha \geq 0 \quad (0.7)$$

$$\left(\frac{\partial W^o}{\partial \beta}\left((1-\alpha)f' - \beta\right)E\left[v'\right] - E\left[v'G^o\right]\right)\Delta\tilde{U}\left(Y^S\right) + E\left[u'G^o\right]\Delta\tilde{V}\left(Y^B\right) \leq 0, \qquad \beta \geq 0 \quad (0.8)$$

$$-E[v']\Delta\tilde{U}\left(Y^S\right) + E[u']\Delta\tilde{V}\left(Y^B\right) = 0 \quad (0.9)$$

I now identify the contract form which is chosen in the bargaining process by investigating the Kuhn-Tucker conditions. Assume first that the contract contains only a fixed payment per unit of groundwater and no positive output share for the wellowner, that is $\alpha = 0$ and $\beta > 0$. Then from (0.7) and (0.9) I have $f' - \beta \le 0$, and from (0.8) and (0.9) I have $\frac{\partial W^o}{\partial \beta}\left(f' - \beta\right) = \frac{E\left[v'G^o\right]}{E\left[v'\right]} - \frac{E\left[u'G^o\right]}{E\left[u'\right]}$. But this is not possible if there is no output sharing between the buyer and the seller. For if $\alpha = 0$ I get from the first-order condition of the seller that $\beta = c$, which in turn implies that the second term on the right hand side of the above condition becomes $E\left[G^o\right]$, whereas for the first term on the right hand side I have $\frac{E\left[v'G^o\right]}{E\left[v'\right]} > E\left[G^o\right]$ because $v'\left(f - \beta G^o - \gamma\right)$ and G^o are positively correlated.[7] From this it follows that the right hand side is strictly positive, whereas the left hand side will never be positive, which is a contradiction. So I must have $\alpha > 0$. Put it in another way, inspection of (0.1) reveals that from $\alpha = 0$ and $\beta = c$ it follows that $Y^S = \gamma + \overline{Y}^S$, which is constant, the requirement that $\beta = c$ arising from the need to ensure that the seller will sell any amount of water, since in this case there are no incentives for the seller arising from an output share. But since the buyer is also risk averse, it cannot be optimal that he bears all the risk alone.

Next consider the case where $\alpha > 0$ and $\beta = 0$. Then from (0.7) and (0.9) I have $(1-\alpha) f' = 0$, and from (0.8) and (0.9) $\frac{E\left[v'G^o\right]}{E\left[v'\right]} \ge \frac{E\left[u'G^o\right]}{E\left[u'\right]}$. That is, I have $\alpha = 1$; but this together with the said weak inequality yields a contradiction, because in this case the left hand

[7] $E\left[\left(v' - E\left[v'\right]\right)\left(G^o - E\left[G^o\right]\right)\right] = E\left[v'G^o\right] - E\left[v'\right]E\left[G^o\right] > 0$.

side of the inequality is equal to $E\left[G^o\right]$ whereas for the right hand side I have $\frac{E\left[u'G^o\right]}{E\left[u'\right]} > E\left[G^o\right]$ because $u'\left(f - cG^o + \gamma\right)$ and G^o are positively correlated. This yields the following proposition:[8]

<u>Proposition 1.</u> *If both the buyer and the seller are risk averse, the amount of groundwater delivered by the seller is observable, but not enforceable, and there are constant marginal costs of groundwater extraction, then the optimal contract is such that* $0 < \alpha^o < 1$ *and* $\beta^o > 0$.

What else can be said about the optimal contract? Consider again the conditions which must hold in the optimum:

$$(1-\alpha^o)f' - \beta^o = 0 \tag{0.10}$$

$$\frac{E\left[v'G^o\right]}{E\left[v'\right]} = \frac{E\left[u'G^o\right]}{E\left[u'\right]} \tag{0.11}$$

together with (0.9) and the first order condition for the wellowner, $\alpha^o f' + \beta^o - c = 0$. (0.10) combined with the first order condition for the seller yields the efficiency condition $f' = c$ which means that as a consequence of the optimal contract the amount of groundwater is chosen by the wellowner in a way that the marginal product equals the marginal cost. This result follows from the fact that for the optimal contract I have $1-\alpha^o = \frac{\beta^o}{c}$, or $(1-\alpha^o)c = \beta^o$: Inserting this result in the state-dependent income equations for the buyer and the seller, I obtain

[8] Notice that $\alpha = 1$ only if $\beta = 0$ because $(1-\alpha)f' - \beta = 0$ for $\alpha > 0$.

$Y_R^B = \left(1-\alpha^o\right)\left\{f - cG^o\right\} - \gamma^o + \overline{Y}^B$ and $Y_R^S = \alpha^o\left\{f - cG^o\right\} + \gamma^o + \overline{Y}^S$, that is, the output produced with the groundwater delivered by the wellowner is shared between the buyer and the seller in the same proportion as the costs incurred by the wellowner in producing the groundwater. This result in turn is caused by the fact that the input in question, namely groundwater, is observable, and can be paid by means of a fixed payment per unit. Also, there is an instrument besides α and β, namely γ, which serves to transfer utility from one party to the other, so this task has not to be shouldered by α and β. Thus, in the theoretical examination of groundwater contracts that contract form turns out to be the optimal one, which is often empirically observed for tenancy contracts, but which in the related theoretical literature is explained by information asymmetries between the landlord and the tenant.[9] In our model, cost-sharing is possible because the input is observable, and it is used to achieve an efficient input supply by the wellowner.

Equation (0.11) states how the risk is shared between the two parties. One can think of $\frac{E\left[v'G^o\right]}{E\left[v'\right]} - E\left[G^o\right]$ and $\frac{E\left[u'G^o\right]}{E\left[u'\right]} - E\left[G^o\right]$ as the buyer's and the seller's risk premiums, respectively, since if both of them were risk neutral, $\frac{E\left[v'G^o\right]}{E\left[v'\right]}$ and $\frac{E\left[u'G^o\right]}{E\left[u'\right]}$ would reduce to $E\left[G^o\right]$. Thus, the condition states that the contract should yield the same risk premium for the two parties. But this in turn implies that the risky income generated under the contract is optimally (efficiently) shared between the two parties. To see why, consider, for example, a contractual arrangement under which $\frac{E\left[v'G^o\right]}{E\left[v'\right]} < \frac{E\left[u'G^o\right]}{E\left[u'\right]}$. This constellation means that an additional marginal unit of risky income will 'cost' the buyer less than the seller, which cannot be

[9] See Braverman and Stiglitz (1986).

optimal. Therefore, it would increase efficiency to transfer risky income from the buyer to the seller until $\frac{E\left[v'G^o\right]}{E\left[v'\right]}=\frac{E\left[u'G^o\right]}{E\left[u'\right]}$. Optimal risk sharing is achieved in this case, because the necessity to transfer risk is unaffected by incentive effects.

<u>Proposition 2.</u> *Under the optimal contract, the output produced with the groundwater is shared among the buyer and the seller in the same proportion as the costs of groundwater extraction. The seller chooses the amount of groundwater such that* $f'\left(R+G(\alpha^o,\beta^o)\right)=c$. *Finally, risk is optimally shared between the two parties.*

From (0.9) and (0.11) it is clear that the particular values of α, β, and γ depend on the parties' degrees of risk aversion. But since nothing more can be said about the characteristics of the solution in general, I now proceed to analyse particular utility functions.

3.2 Exponential utility

Assume that both the buyer and the seller have a utility function of the CARA (constant absolute risk aversion) form, $v\left(Y^B\right)=-e^{-aY^B}$ and $u\left(Y^S\right)=-e^{-aY^S}$ respectively, where a is the coefficient of absolute risk aversion which for the beginning is assumed to be the same for both parties. Now, from (0.10), (0.11) and the first order condition of the seller, I can derive

$$\frac{\int_0^{\bar{R}}\exp\left[-a\beta R\right]Rh\left(R\right)dR}{\int_0^{\bar{R}}\exp\left[-a\beta R\right]h\left(R\right)dR}=\frac{\int_0^{\bar{R}}\exp\left[a\left(\beta-c\right)R\right]Rh\left(R\right)dR}{\int_0^{\bar{R}}\exp\left[a\left(\beta-c\right)R\right]h\left(R\right)dR}. \tag{0.12}$$

From (0.12), proposition 3 follows immediately, on condition that the solution to (0.12) is unique.[10]

<u>Proposition 3.</u> *If the buyer and the seller have identical utility functions satisfying CARA, the optimal contract is such that*

$$\alpha^o = \frac{1}{2}, \qquad \beta^o = \frac{c}{2}.$$

Notice, that this result is independent of the distribution of the amount of rainfall. Since there are no wealth effects by virtue of CARA, the parameters which determine how the risk is shared under the optimal contract are independent of the risky income (and its determinants) realized under the contract. However, not independent of the rainfall distribution is the unrestricted transfer payment. Using (0.9) and the results from proposition 3, I can derive

<u>Proposition 3a.</u> *If the buyer and the seller have identical utility functions satisfying CARA, the optimal unrestricted transfer payment is*

$$\gamma^o = \frac{1}{2a} \ln\left(\int_0^{\bar{R}} \exp\left[-af\left(R\right)\right] h\left(R\right) dR \right),$$

where $\int_0^{\bar{R}} \exp\left[-af\left(R\right)\right] h\left(R\right) dR$ *is the risky part of the buyer's expected disagreement payoff.*

Notice, that the optimal unrestricted transfer payment is not chosen to equalize the gains from cooperation, i.e. to achieve $\Delta\tilde{V}\left(Y^B\right) = \Delta\tilde{U}\left(Y^S\right)$, but it is chosen to equalize the gains from

[10] Conditions for the solution to (0.11) to be unique are stated in the appendix.

cooperation weighted by the marginal expected utilities, i.e. $E[v']\Delta\tilde{U}(Y^S) = E[u']\Delta\tilde{V}(Y^B)$, since both parties are risk averse. Thus, the first-best outcome where $\Delta\tilde{V}(Y^B) = \Delta\tilde{U}(Y^S)$ is not achieved by the contract due to the risk aversion of the parties.

Using the results from propositions 3 and 3a, the state-dependent incomes of the buyer and the seller under the optimal contract are:

$$Y_R^B = \frac{1}{2}\left[f - cG^o\right] - \frac{1}{2a}\ln\left[\int_0^{\bar{R}} \exp\left[-af(R)\right]h(R)\,dR\right] + \bar{Y}^B$$

$$Y_R^S = \frac{1}{2}\left[f - cG^o\right] + \frac{1}{2a}\ln\left[\int_0^{\bar{R}} \exp\left[-af(R)\right]h(R)\,dR\right] + \bar{Y}^S .$$

That is, in the case of identical CARA utility functions, the output and the costs for the input in question are shared equally between the buyer and the seller, and as a consequence the seller delivers the efficient amount of groundwater. Now consider the expression for the optimal transfer payment, which can be positive or negative, and depends on the degree of risk aversion and on the risky part of the disagreement payoff of the buyer. The sign of this transfer payment will depend on whether $\int_0^{\bar{R}} \exp\left[-af(R)\right]h(R)\,dR = \int_0^{\bar{R}} \left|-\exp\left[-af(R)\right]\right| h(R)\,dR \gtreqless 1$. The bigger the absolute value of this expression, the smaller is the buyer's expected utility in the alternative case where he has no contract with the seller. That is, the smaller the buyer's alternative expected utility, the higher is the transfer payment he has to make to the seller, or the smaller is the transfer payment he receives from the seller, depending on whether the absolute value of his alternative expected utility is bigger or smaller than one. If this alternative expected utility is equal to one, there will be no transfer payment. Considering the effect of change of the degree of risk aversion a on the transfer payment, thereby neglecting the indirect effect which is caused by changes in the expected disagreement payoff of the buyer, I find that the absolute values of the transfer payments are the smaller the higher the degrees of risk aversion of the contracting parties. This can be interpreted as a risk premium which each party is willing to pay in order to avoid to bear all risk alone. The said risk premium is increasing in the degree of risk aversion.

To see how particular parameters influence the transfer payment, consider some comparative static results:

$$\frac{\partial \gamma^o}{\partial a} = -\frac{1}{2a^2}\ln\left[\int_0^{\bar{R}} \exp\left[-af\left(R\right)\right]h\left(R\right)dR\right] - \frac{1}{2a}\frac{\int_0^{\bar{R}} f\left(R\right)\exp\left[-af\left(R\right)\right]h\left(R\right)dR}{\int_0^{\bar{R}} \exp\left[-af\left(R\right)\right]h\left(R\right)dR}$$

The second term in this expression is always bigger than zero, whereas the sign of the first term again depends on whether the buyer's alternative expected utility is smaller or bigger than one in absolute terms. The smaller the buyer's alternative expected utility is, the more likely is the first term to be negative, and therefore the more likely is γ^o to be decreasing in *a*. The first term is the direct 'risk premium' effect of *a* on γ^o and the second term is the indirect effect which arises, because the degree of absolute risk aversion has a positive influence on the buyer's alternative expected utility.

To investigate the effect of a change in the rainfall distribution, assume, for example, that the amount of rainfall is uniformly distributed on the interval $\left[0,\bar{R}\right]$ which implies that the density function is $h\left(R\right)=\frac{1}{\bar{R}}$. Differentiating γ^o with respect to $\bar{R}$ I have

$$\frac{\partial \gamma^o}{\partial \bar{R}} = \frac{1}{2a}\frac{1}{\bar{R}}\left\{-1+\frac{\exp\left[-af\left(\bar{R}\right)\right]}{\int_0^{\bar{R}} \exp\left[-af\left(R\right)\right]\frac{1}{\bar{R}}dR}\right\} > 0 \text{, since } \exp\left[-af\left(\bar{R}\right)\right] - \int_0^{\bar{R}} \exp\left[-af\left(R\right)\right]\frac{1}{\bar{R}}dR > 0 .$$

Thus, for a rainfall distribution with higher mean and higher variance which first-order stochastically dominates the other, the buyer has to make a higher transfer payment to the seller, or receives a smaller transfer payment from the latter, depending on whether γ^o is positive or negative. That is, a change in the distribution which makes the buyer better off in the case of no contract, results in a higher transfer payment from the buyer to the seller under the contract. This somewhat puzzling result is a consequence of the maximization of the product of the gains from cooperation.

Now consider the more realistic case in which the utility functions still exhibit CARA, but the buyer and the seller have different coefficients of absolute risk aversion. Let the utility functions be $v(Y^B) = -e^{-aY^B}$ and $u(Y^S) = -e^{-bY^S}$, respectively, where a and b are the coefficients of absolute risk aversion. Then, again, from (0.9), (0.10), (0.11), and the first order condition for the seller I can derive the following result:

<u>Proposition 4.</u> *For CARA and different degrees of absolute risk aversion the optimal contract is such that*

$$\alpha^o = \frac{a}{a+b}, \qquad \beta^o = \frac{b}{a+b}c.$$

The intuition behind proposition 4, as well as behind propositions 3 and 3a, is, that the optimal contract requires production efficiency, that is $f'(W^o) = c$, and that, taking into account the risk aversion of the two parties, α and β must be chosen accordingly. Therefore, from proposition 4 and (0.2), I have $\frac{a}{a+b}f' + \frac{bc}{a+b} = c$, as required.

Notice that in this little more complicated setting an explicit solution for γ cannot be derived. However, the expressions for α and β in the case of different degrees of risk aversion give a more detailed insight in the nature of the optimal contract than in the case of identical degrees of risk aversion. Consider first the expression for the optimal output share: The higher the degree of risk aversion of the buyer and the smaller the degree of risk aversion of the seller, the higher is the output share which the seller receives. At the same time, the fixed payment per unit of groundwater paid by the buyer is decreasing in the degree of risk aversion of the buyer and increasing in the degree of risk aversion of the seller. In general, the optimal contract is such that the less risk averse party bears a larger proportion of the costs of the ex ante random groundwater production while, ex post, the output is made certain by the wellowner's choice in the third stage. If the wellowner is risk neutral and the farmer is risk averse, the wellowner receives the entire crop output, carries all the costs of producing the groundwater, and there is a fixed payment between the wellowner and the farmer which serves as an instrument to transfer utility. In this case the transfer payment γ must have a negative sign because otherwise the contract would not be attractive for the buyer. If on the other hand the farmer is risk neutral, and the wellowner is risk averse, the farmer receives the entire crop output and carries all the costs of producing the groundwater. Again utility is transferred through the fixed payment which now must have a positive sign.

Given the optimal contractual parameters in the setting with CARA and different degrees of absolute risk aversion, the state-dependent incomes for the buyer and the seller are:

$$Y_R^B = \frac{b}{a+b}\left(f - cG^o\right) - \gamma^o + \overline{Y}^B$$

and

$$Y_R^S = \frac{a}{a+b}\left(f - cG^o\right) + \gamma^o + \overline{Y}^S.$$

That is, in each state of nature the risky surplus is shared between the wellowner and the farmer in a proportion which reflects their respective degrees of absolute risk aversion: if the seller is less risk averse than the buyer, the seller receives a larger share of the risky surplus than the buyer and vice versa. Reallocations of utility resulting from the risk-sharing arrangement are dealt with by the unrestricted transfer payment.

3.3 Logarithmic utility

As I have shown above, in the case of CARA the cropshare depends only on the degrees of risk aversion of the contracting parties. Now consider the case of a utility function which exhibits constant relative risk aversion (CRRA) for both the seller and the buyer. More specifically, I assume that $v\left(Y^B\right) = \ln Y^B$ and $u\left(Y^S\right) = \ln Y^S$. In this setting too, propositions 1 and 2 remain valid and I only have to reconsider expression (0.11), which states how the risk is shared between the contracting parties. Again assuming a uniform distribution of R on $\left[0, \overline{R}\right]$, I can rewrite (0.11) as

$$\frac{\int_0^{\overline{R}} v'\left(Y^B\right) G^o \frac{1}{\overline{R}} dR}{\int_0^{\overline{R}} v'\left(Y^B\right) \frac{1}{\overline{R}} dR} = \frac{\int_0^{\overline{R}} u'\left(Y^S\right) G^o \frac{1}{\overline{R}} dR}{\int_0^{\overline{R}} u'\left(Y^S\right) \frac{1}{\overline{R}} dR},$$

which can be simplified to

$$\frac{\int_0^{\overline{R}} v'\left(Y^B\right) R dR}{\int_0^{\overline{R}} v'\left(Y^B\right) dR} = \frac{\int_0^{\overline{R}} u'\left(Y^S\right) R dR}{\int_0^{\overline{R}} u'\left(Y^S\right) dR}.$$

Using logarithmic utility, integrating, and rearranging terms gives

$$\frac{\beta\left(\alpha f+\gamma+\overline{Y}^S\right)+\left(\beta-c\right)\left((1-\alpha)f-\gamma+\overline{Y}^B\right)}{\left(\beta-c\right)\beta}=$$

$$\overline{R}\frac{\ln\left(\dfrac{\alpha f+(\beta-c)G^0+\gamma+\overline{Y}^S}{\alpha f+(\beta-c)(G^0-\overline{R})+\gamma+\overline{Y}^S}\dfrac{(1-\alpha)f-\beta(G^0-\overline{R})-\gamma+\overline{Y}^B}{(1-\alpha)f-\beta G^0-\gamma+\overline{Y}^B}\right)}{\ln\left(\dfrac{\alpha f+(\beta-c)G^0+\gamma+\overline{Y}^S}{\alpha f+(\beta-c)(G^0-\overline{R})+\gamma+\overline{Y}^S}\right)\ln\left(\dfrac{(1-\alpha)f-\beta(G^0-\overline{R})-\gamma+\overline{Y}^B}{(1-\alpha)f-\beta G^0-\gamma+\overline{Y}^B}\right)}. \quad (0.13)$$

One solution of (0.13) is characterized by the following condition:

$$\left(\beta-(1-\alpha)c\right)f+\gamma c+\beta\overline{Y}^S+(\beta-c)\overline{Y}^B=0 \quad (0.14)$$

because, if equation (0.14) holds, the numerators of both the left hand side and the right hand side of equation (0.13) vanish. Since, in the optimal contract, I have $\beta^o=(1-\alpha^o)c$, from (0.14) I can derive $\gamma^o=\alpha^o\overline{Y}^B-(1-\alpha^o)\overline{Y}^S$. This together with the state-dependent incomes for the buyer and the seller leads to

<u>Proposition 5.</u> *If the buyer and the seller have identical logarithmic utility functions and the amount of rainfall is uniformly distributed on* $\left[0,\overline{R}\right]$*, the optimal contract is such that both the buyer and the seller receive a prespecified share of the total income which is generated under the terms of the contract in each state of nature, that is,* $Y^{Bo}=(1-\alpha^o)Y^o$ *and* $Y^{So}=\alpha^oY^o$*, where* $Y^o=f-cG^o+\overline{Y}^B+\overline{Y}^S$.

This result is in contrast to the findings of Propositions 3 and 4 where the optimal values of the cropshare and of the fixed payment per unit groundwater were determined independently of the transfer payment. The cropshare and the costshare depended only on the degrees of risk aversion; all other factors which play a role in the bargaining over the contract terms were captured by the transfer payment. In the special case considered in this section, in contrast, the cropshare and the costshare cannot be determined independently of the transfer payment, from which it follows that, in this setting, I will be able to explain how the cropshare depends on the certain incomes and the distribution of the rainfall. Since I consider only linear contracts, the contractual parameters α, β, and γ are not allowed to vary with the state-dependent incomes of the buyer and the seller, which is the reason why both risk averse parties receive fixed shares of the random cake.

Since $Y^o = Y^{Bo} + Y^{So}$ and since, in equilibrium, the coefficients of absolute risk aversion of the buyer and the seller are in this special case $RA^B\left(Y^{B^o}\right) = \frac{1}{Y^{B^o}}$ and $RA^S\left(Y^{S^o}\right) = \frac{1}{Y^{S^o}}$, respectively, I can write $\alpha^o = \frac{RA^B\left(Y^{Bo}\right)}{RA^B\left(Y^{Bo}\right) + RA^S\left(Y^{So}\right)}$. Strictly speaking, this is a tautology, stating for example that, if the cropshare is close to unity, the state-dependent income of the buyer is very small, which in turn implies that his coefficient of absolute risk aversion is very high whereas the state-dependent income of the seller is relatively high, and his coefficient of absolute risk aversion is rather small, from which it follows that the right hand side of the expression is also near to 1. One can see from the above expression that also in the case of logarithmic utility functions the seller's optimal cropshare depends on the degrees of absolute risk aversion of the contracting parties, but since in this case risk aversion varies with income, the cropshare is not independent of factors which determine the conditions under which the contract is made.

To characterize the solution for α, I substitute the results of proposition 5 for the state-dependent incomes of the buyer and the seller into the first-order condition for γ to obtain

$$\frac{\alpha^o}{1-\alpha^o}=\frac{\ln\left(1-\alpha^o\right)+\int_0^{\overline{R}}\ln Y^o\frac{1}{\overline{R}}dR-\int_0^{\overline{R}}\ln\left(f(R)+\overline{Y}^B\right)\frac{1}{\overline{R}}dR}{\ln\alpha^o+\int_0^{\overline{R}}\ln Y^o\frac{1}{\overline{R}}dR-\ln\overline{Y}^S} \tag{0.15}$$

or

$$\frac{\alpha^o}{1-\alpha^o}=\frac{E\left[v\left(Y^{B^o}\right)\right]-E\left[v\left(f\left(R\right)+\overline{Y}^B\right)\right]}{E\left[u\left(Y^{S^o}\right)\right]-E\left[u\left(\overline{Y}^S\right)\right]}=\frac{\Delta\tilde{V}\left(Y^{B^o}\right)}{\Delta\tilde{U}\left(Y^{S^o}\right)}$$

Since there is no explicit solution for α^o, I use comparative statics to identify the factors on which the optimal value of α depends. Notice, first, that it is obvious from equation (0.15) that - in spite of the fact that I have assumed identical utility functions for the buyer and the seller - α is not equal to one half unless both parties' disagreement payoffs are equal, which is not surprising, since γ is no longer a pure transfer instrument.

The following comparative static results with respect to c, $\overline{R}$, $\overline{Y}^B$, and $\overline{Y}^S$ can be obtained:

$$\frac{\partial\alpha}{\partial c}=\frac{(1-2\alpha)E\left[\frac{G^o}{Y^o}\right]}{\det}\lesseqgtr 0 \qquad \Leftrightarrow \qquad \frac{1}{2}\lesseqgtr\alpha \tag{0.16}$$

$$\frac{\partial\alpha}{\partial\overline{R}}=\frac{\frac{1}{\overline{R}}\left\{\alpha\left[E\left[u\left(Y^{S^o}\right)\right]-u\left(Y^{S^o}\left(\overline{R}\right)\right)\right]-\left(1-\alpha\right)\left[E\left[v\left(Y^{B^o}\right)\right]-v\left(Y^{B^o}\left(\overline{R}\right)\right)\right]+\left(1-\alpha\right)\left[E\left[v\left(f\left(R\right)+\overline{Y}^B\right)\right]-v\left(f\left(R\right)+\overline{Y}^B\right)\right]\right\}}{\det} \tag{0.17}$$

$$\frac{\partial\alpha}{\partial\overline{Y}^B}=\frac{\left(1-\alpha\right)^2E\left[v'\left(Y^{B^o}\right)\right]-\alpha^2E\left[u'\left(Y^{S^o}\right)\right]-\left(1-\alpha\right)E\left[v'\left(f\left(R\right)+\overline{Y}^B\right)\right]}{\det}<0 \tag{0.18}$$

(Proof see appendix.)

$$\frac{\partial \alpha}{\partial \bar{Y}^S} = \frac{(1-\alpha)^2 E\left[v'\left(Y^{B^o}\right)\right] - \alpha^2 E\left[u'\left(Y^{S^o}\right)\right] + \alpha E\left[u'\left(\bar{Y}^S\right)\right]}{\det} > 0 \tag{0.19}$$

(Proof see appendix.),

where $\det = \Delta\tilde{U}\left(Y^{S^o}\right) + \Delta\tilde{V}\left(Y^{B^o}\right) + \alpha E\left[u'\left(Y^{S^o}\right)Y^o\right] + (1-\alpha)E\left[v'\left(Y^{B^o}\right)Y^o\right] > 0$ is the determinant of the Hessian of the related optimization problem.

At first glance, I can only say that (0.16) and (0.17) may have either sign depending on the optimal value of α. An interesting value of α at which to evaluate the above derivatives is $\alpha = \frac{1}{2}$. It was said above, that this is an unlikely optimal value for α since this will hold only if both parties have the same disagreement payoff. But since $\alpha = \frac{1}{2}$ is the value of the seller's cropshare which is the most frequently observed in real world contracts, I think that one can gain some further insights into the nature of groundwater contracts by using this evaluation point. Then I have:

$$\left.\frac{\partial \alpha}{\partial c}\right|_{\alpha=0.5} = 0,$$

$$\left.\frac{\partial \alpha}{\partial \bar{R}}\right|_{\alpha=0.5} = \frac{\frac{1}{2\bar{R}}\left\{E\left[v\left(f(R)+\bar{Y}^B\right] - v\left(f(\bar{R})+\bar{Y}^B\right)\right\}}{\det} < 0,$$

Further, since $\beta^o = (1-\alpha^o)c$, I have

$$\left.\frac{\partial \beta}{\partial c}\right|_{\alpha=0.5} = \frac{1}{2} > 0,$$

$$\left.\frac{\partial \beta}{\partial \bar{R}}\right|_{\alpha=0.5} = -\left.\frac{\partial \alpha}{\partial \bar{R}}\right|_{\alpha=0.5} c > 0,$$

$$\left.\frac{\partial \beta}{\partial \overline{Y}^{B}}\right|_{0\le\alpha\le1} > 0 \text{ and}$$

$$\left.\frac{\partial \beta}{\partial \overline{Y}^{S}}\right|_{0\le\alpha\le1} < 0 .$$

I can summarize these results in the following proposition.

<u>Proposition 6.</u> *Consider a situation in which the output is shared equally between the buyer and the seller, and in which the rainfall is uniformly distributed on the interval* $\left[0,\overline{R}\right]$. *Then the seller's optimal cropshare is*

(i) decreasing in the buyer's certain income,

(ii) increasing in the seller's certain income,

(iii) decreasing in the upper bound of the rainfall distribution,

and the optimal fixed payment per unit groundwater is

(i) increasing in the buyer's certain income,

(ii) decreasing in the seller's certain income,

(iii) increasing in the upper bound of the rainfall distribution.

As far as $\frac{\partial \alpha}{\partial \overline{Y}^{S}}$ and $\frac{\partial \alpha}{\partial \overline{Y}^{B}}$ are concerned, the above results hold for all cropshares. For $\frac{\partial \alpha}{\partial \overline{R}}$, it is easy to see that for $\frac{1}{2} \le \alpha \le 1$, $\frac{\partial \alpha}{\partial \overline{R}} < 0$, whereas for $0 \le \alpha < \frac{1}{2}$ the sign of the numerator is not clear. Consider the extreme case where $\alpha = 0$. Then

$$\frac{\partial \alpha}{\partial \overline{R}} = \frac{\frac{1}{\overline{R}}\left\{E\left[v\left(f\left(R\right)+\overline{Y}^{B}\right)\right]-v\left(f\left(\overline{R}\right)+\overline{Y}^{B}\right)\right\}-\frac{1}{\overline{R}}\left\{E\left[v\left(Y^{o}\right)\right]-v\left(Y^{o}\left(\overline{R}\right)\right)\right\}}{\det} .$$

This expression has a negative sign if

$$\left|E\left[v\left(f\left(R\right)+\bar{Y}^{B}\right)\right]-v\left(f\left(\bar{R}\right)+\bar{Y}^{B}\right)\right|>\left|E\left[v\left(Y^{o}\right)\right]-v\left(Y^{o}\left(\bar{R}\right)\right)\right|.$$

That is, if the distribution of the total income generated under the optimal contract is such that the absolute difference between the expected utility of this income and the utility of the contract income at the upper bound of the rainfall distribution is smaller than the absolute difference between the expected utility of the buyer's non-contract income and the utility of this income at the upper bound of the rainfall distribution, then the seller's cropshare is decreasing in the upper bound of the rainfall distribution for all possible values of α.

How do these results compare to empirical observation? Concerning the amount of rainfall, in areas with a relatively high annual amount of rainfall, sharecropping contracts for groundwater transactions are rare, or non-existent, while the selling of groundwater against a fixed rate per unit is very common. On the other hand, in areas with relatively little rainfall over the year, farmers frequently rely on the use of share contracts as a mode of payment for groundwater transactions. In these areas, fixed rate payments are used only in a few cases, especially if the buyer is in need of only a small number of irrigations. As it was mentioned in section 2, it seems that this pattern can be explained empirically by the risk aversion and/or the wealth-constrainedness of the farmers.

To summarize, I find that, in a model with two risk averse parties, an observable but not enforceable input and constant marginal costs for this input, the outcome implies an efficient supply of groundwater to the farmer's field. This is a consequence of the observability of the crucial input, namely groundwater, because in the case of a perfectly observable input there are three instruments which can be used in a linear contract: an output share, a fixed payment per each unit of the input, and an unrestricted transfer payment. The combination of an output share and a fixed payment per unit of the input allows the choice of a contract in which the costs of extracting groundwater are shared in the same proportion as the output produced with this groundwater. As a consequence, the wellowner chooses the efficient amount of groundwater. Since, by assumption, I ruled out corner solutions for the amount of groundwater due to too much rainfall, all natural uncertainty is resolved by the time the decision to supply groundwater must be made. In this sense, there is never any 'regret' about G^o, only about (α, β, γ).

3.4 The case of only two contractual parameters

An unrestricted transfer payment is not observed in real world contracts, at least not in an explicit monetary form. It is possible that such transfers are made in a different way, for example, by one party delivering services such as field labour or draught power to the other[11]; but transactions of this kind are rather seldom reported. As mentioned above, the existence of a transfer payment is crucial for the result that the optimal parameter values of α and β depend on only the degrees of risk aversion of the seller and the buyer and do not contain any terms which are related to the provision of incentives or the need to transfer utility. Since, in reality, I observe no pure transfer payments, it is of some interest to characterize the solution for the model given if the set of contractual parameters chosen in the bargaining process is limited to a cropshare and a fixed payment per unit.

Setting γ equal to zero in the income equations for the buyer and the seller, I can employ the model developed above without other modifications. Again the product of the utility differences is maximised, but this time only with respect to α and β:

[11] This services can be lump-sum transfers. It could be negotiated at the beginning of the season, for example, that one party may use the bullock pair of the other party for ten days.

$$\max_{\alpha,\beta} \Delta\tilde{V}\left(Y^B\right)\Delta\tilde{U}\left(Y^S\right) \tag{0.20}$$

$$\text{s.t.} \qquad \alpha \in [0,1], \qquad \beta \geq 0,$$

which yields the first-order conditions

$$E[v']\left(-f + \frac{\partial G}{\partial \alpha}\big((1-\alpha)f_W - \beta\big)\right)\Delta\tilde{U}\left(Y^S\right) + E[u']f\Delta\tilde{V}\left(Y^B\right) \leq 0, \quad \alpha \geq 0 \tag{0.21}$$

$$\left\{E[v']\frac{\partial G}{\partial \beta}\big((1-\alpha)f_W - \beta\big) - E[v'G]\right\}\Delta\tilde{U}\left(Y^S\right) + E[u'G]\Delta\tilde{V}\left(Y^B\right) \leq 0, \quad \beta \geq 0 \tag{0.22}$$

Consider first the case where $\alpha = 0$ and $\beta > 0$. From the seller's first-order condition I then have $\beta = c$, which in turn implies that $\Delta\tilde{U}\left(Y^s\right) = 0$. This cannot be optimal because every contract which splits the entire surplus in a way that both parties gain would yield a higher value of the product of the utility differences. Now assume that $\alpha > 0$ and $\beta = 0$. Then from (0.21) and (0.22) I have

$$\frac{\left(\frac{\partial G}{\partial \beta}(1-\alpha)f_W - E[G]\right)f}{\frac{\partial G}{\partial \alpha}(1-\alpha)f_W - f} \geq \frac{E[u'G]}{E[u']}.$$

Since, in this case, u' and G are positive correlated, the inequality $\frac{E[u'G]}{E[u']} > E[G]$ holds[12]. Now consider the condition

[12] $E\left[\left(u' - E[u']\right)\left(G - E[G]\right)\right] = E[u'G] - E[u']E[G] > 0$.

$$\frac{\left(\frac{\partial G}{\partial \beta}(1-\alpha)f_W - E[G]\right)f}{\frac{\partial G}{\partial \alpha}(1-\alpha)f_W - f} \geq E[G],$$

which would characterize the solution for a risk neutral seller. Substituting $\frac{\partial G}{\partial \alpha}$ and $\frac{\partial G}{\partial \beta}$ from above, I get, after some manipulation, $f_W \geq \frac{f}{E[G]}$, which is a contradiction because of the concavity of the production function. Moreover, if I have a contradiction for a risk neutral seller, I also have a contradiction for a risk averse seller because $\frac{E[u'G]}{E[u']} > E[G]$. It follows that the optimal contract contains a positive cropshare and a fixed payment per unit. In this case, from (0.21) and (0.22) I have

$$\frac{E[v']\frac{\partial G}{\partial \beta}\big((1-\alpha)f_W - \beta\big) - E[v'G]}{E[v']\frac{\partial G}{\partial \alpha}\big((1-\alpha)f_W - \beta\big) - E[v']f} = \frac{E[u'G]}{E[u']f}. \tag{0.23}$$

Equation (0.23) shows that in the absence of a fixed transfer payment, the other two contractual parameters depend on the terms reflecting the incentive effects of the parameters if an efficient use of water cannot be guaranteed.[13] If, on the other hand, incentive problems are not to taken into account, i.e. if an efficient choice of the amount of groundwater by the wellowner could be enforced, equation (0.23) reduces to $\frac{E[v'G]}{E[v']} = \frac{E[u'G]}{E[u']}$, which is the same optimality condition as in the model with a fixed transfer payment. In general, however, the equation $(1-\alpha)f_W - \beta = 0$ does not hold in the case of only two contractual parameters, from which it follows that the

[13] Efficiency would imply that $(1-\alpha)f_W - \beta = 0$.

solution for the cropshare and for the fixed payment per unit in this case will deviate from the solution in the case of three contractual parameters.[14]

4 Conclusion

The aim of this paper was to find an explanation for the fact that in some of the informal groundwater transactions observed in south-Indian villages the wellowner receives a share of the crop output in exchange for the groundwater delivered by him, whereas in other transactions the farmer keeps the entire crop output, and pays a fixed amount for each unit of groundwater the wellowner delivers. In a bargaining framework I showed that – depending on the shape of the utility functions of the buyer and the seller - important determinants of the choice of the contract form are the degrees of absolute risk aversion of the contracting parties in the case of exponential utility and factors that influence the contract-dependent as well as the contract-independent income of the parties, such as the distribution of the amount of rainfall, the marginal costs of groundwater extraction and the certain incomes of the buyer and the seller. Our results underline the fact that sharecropping arrangements in the context of informal groundwater markets, which in the empirical literature dealing with this phenomenon are often condemned as exploitative, are a powerful instrument to overcome inefficiencies which arise in a risky environment when there are incomplete or missing markets for the allocation of a scarce resource and for the allocation of risk, an imperfect water market, or a missing insurance market on the village level. Our results also show, that under certain conditions even productive efficiency can be achieved by the use of groundwater share contracts.

[14] This is the usual story in the literature on tenancy contracts when a full division of the not perfectly observable labour among instruments cannot be attained.

The analysis of this paper is an important first step in understanding the choice of contracts in an agricultural economy where water is scarce and opens an entire research agenda in the field of agricultural contracts. Future research has to address questions such as how the allocation of groundwater is related to the allocation of land. How does the availability of irrigation water influence a farmer's decision to rent in or rent out land? What are the effects of an increasing density of wells in a village on the contract form chosen? How can the possibility of an overexploitation of the local groundwater resources be taken into account? What are the effects of wealth or credit constraints on the side of the water buyer? Most closely related to the research of this paper is the question how the timing of the farmer's decision on when to approach a wellowner for additional irrigation water can be endogenized and how this matters for the contract choice and for the efficiency achieved by the contract. The farmer's decision how long to wait before entering a contract potentially depends on his beliefs about the quantity of rainfall during the remainder of the growing season. If he decides to wait for future rainfall without entering a contract he has to take into account that the condition of his crop may worsen in the meantime because of a lack of water, which may negatively influence his future bargaining power. This timing consideration plays no role in the context of tenancy contracts, and addressing it may help to deepen the general understanding of how and under which circumstances share contracts can balance risk sharing considerations and incentive problems when markets are incomplete or missing.

Appendix

Conditions for a unique solution to (0.11):

Using the condition which must hold in the optimum, $\left(1-\alpha\right)=\frac{\beta}{c}$, I can write the state-dependent incomes of the buyer and the seller under the optimal contract as

$$Y^B=\beta\left(\frac{f}{c}-G^o\right)-\gamma+\bar{Y}^B \quad \text{and} \tag{A.1}$$

$$Y^S=\left(\beta-c\right)\left(-\frac{f}{c}+G^o\right)+\gamma+\bar{Y}^S. \tag{A.2}$$

Employing the envelope theorem, the derivatives of Y^B and Y^S with respect to β are

$$\frac{\partial Y^B}{\partial\beta}=\frac{f}{c}-G^o \gtreqless 0 \quad \Leftrightarrow \begin{cases} DRS \\ CRS \\ IRS \end{cases} \quad \text{and} \tag{A.3}$$

$$\frac{\partial Y^S}{\partial\beta}=-\frac{f}{c}+G^o \gtreqless 0 \quad \Leftrightarrow \begin{cases} IRS \\ CRS \\ DRS \end{cases}. \tag{A.4}$$

In the case of CARA, (0.11) can be shown to depend only on β. The solution for β will be unique, if the left-hand side of (0.11) is strictly increasing in β and the right-hand side is strictly decreasing in β, or vice versa.

$$\frac{\partial}{\partial\beta}\left(\frac{E\left[v'G^o\right]}{E\left[v'\right]}\right)\gtreqless 0 \quad \Leftrightarrow \quad E\left[v'\right]E\left[v''\frac{\partial Y^B}{\partial\beta}G^o\right]-E\left[v'G^o\right]E\left[v''\frac{\partial Y^B}{\partial\beta}\right]+E\left[v'\right]^2\frac{\partial W^o}{\partial\beta}\gtreqless 0 \tag{A.5}$$

If I assume that the indirect effect, $E[v']^2 \frac{\partial W^o}{\partial \beta} > 0$, which arises because a change in β influences the seller's groundwater decision, can be neglected, then the sign of (A.5) depends on whether $E[v']E\left[v''\frac{\partial Y^B}{\partial \beta}G^o\right] - E[v'G]E\left[v''\frac{\partial Y^B}{\partial \beta}\right] \gtreqless 0$. This can be rewritten using (A.3) as

$$\frac{f}{c}\left(E[v']E\left[v''G^o\right] - E\left[v'G^o\right]E[v'']\right) + E\left[v'G^o\right]E\left[v''G^o\right] - E[v']E\left[v''\left(G^o\right)^2\right] \gtreqless 0 \qquad \text{(A.6)}$$

The first term of (A.6) is equal to zero, because for CARA $\frac{E\left[v'G^o\right]}{E[v']} = \frac{E\left[v''G^o\right]}{E[v'']}$. Therefore,

$$\frac{\partial}{\partial \beta}\left(\frac{E\left[v'G^o\right]}{E[v']}\right) \gtreqless 0 \quad \Leftrightarrow \quad \frac{E\left[v'G^o\right]}{E[v']} \lesseqgtr \frac{E\left[v''\left(G^o\right)^2\right]}{E\left[v''G^o\right]}. \qquad \text{(A.7)}$$

Since except for CRS, (A.3) and (A.4) will always have the opposite sign, for the risk premium of the seller I have

$$\frac{\partial}{\partial \beta}\left(\frac{E\left[u'G^o\right]}{E[u']}\right) \lesseqgtr 0 \quad \Leftrightarrow \quad \frac{E\left[u'G^o\right]}{E[u']} \lesseqgtr \frac{E\left[u''\left(G^o\right)^2\right]}{E\left[u''G^o\right]}. \qquad \text{(A.8)}$$

Thus, if $\frac{E\left[v'G^o\right]}{E[v']} < \frac{E\left[v''\left(G^o\right)^2\right]}{E\left[v''G^o\right]} \quad \forall Y^B$ and $\frac{E\left[u'G^o\right]}{E[u']} < \frac{E\left[u''\left(G^o\right)^2\right]}{E\left[u''G^o\right]} \quad \forall Y^S$, or if $\frac{E\left[v'G^o\right]}{E[v']} > \frac{E\left[v''\left(G^o\right)^2\right]}{E\left[v''G^o\right]} \ \forall Y^B$ and $\frac{E\left[u'G^o\right]}{E[u']} > \frac{E\left[u''\left(G^o\right)^2\right]}{E\left[u''G^o\right]} \ \forall Y^S$, then the solution to (0.11) in the case of CARA is unique.

Determination of the signs of (0.18) and (0.19):

For $\alpha \geq \frac{1}{2}$, $\frac{\partial \alpha}{\partial \bar{Y}^B} < 0$ because $(1-\alpha)^2 E\left[v'\left(Y^{B^o}\right)\right] - \alpha^2 E\left[u'\left(Y^{S^o}\right)\right] < 0$. For the extreme case that $\alpha = 0$,

$$\frac{\partial \alpha}{\partial \bar{Y}^B} = \frac{\int_0^{\bar{R}} \frac{1}{Y^o} \frac{1}{\bar{R}} dR - \int_0^{\bar{R}} \frac{1}{f(R) + \bar{Y}^B} \frac{1}{\bar{R}} dR}{\det} < 0$$

because $E\left[u'\left(Y^o\right)\right] = E\left[v'\left(Y^o\right)\right] < E\left[v'\left(f(R) + \bar{Y}^B\right)\right]$. Therefore it follows that for $0 < \alpha < \frac{1}{2}$ $\frac{\partial \alpha}{\partial \bar{Y}^B} < 0$, too.

Likewise, for $\alpha \leq \frac{1}{2}$, $\frac{\partial \alpha}{\partial \bar{Y}^S} > 0$ because $(1-\alpha)^2 E\left[v'\left(Y^{B^o}\right)\right] - \alpha^2 E\left[u'\left(Y^{S^o}\right)\right] > 0$. For the extreme case that $\alpha = 1$,

$$\frac{\partial \alpha}{\partial \bar{Y}^S} = \frac{\int_0^{\bar{R}} \frac{1}{\bar{Y}^S} \frac{1}{\bar{R}} dR - \int_0^{\bar{R}} \frac{1}{Y^o} \frac{1}{\bar{R}} dR}{\det} > 0$$

because $E\left[u'\left(Y^o\right)\right] = E\left[v'\left(Y^o\right)\right] < E\left[u'\left(\bar{Y}^S\right)\right]$. Therefore it follows that for $\frac{1}{2} < \alpha < 1$ $\frac{\partial \alpha}{\partial \bar{Y}^S} > 0$, too.

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